Product Differentiation Impact on Games Theory Models

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Abstract

Imperfect competition represents one of the main topic of modern economic analysis and it can be easily identified in the current economic climate. Whether we are reffering to the aggregate economic field or to a specific industry, certain topics can be highlighted due to their strong impact on the market analyse tools selection process: the differences between price and quantity competition scenario, competitors movement timing in a duopoly game, the role of product differentiation in determining market output/price levels or the presence of the market hierarchical structure. In order to better expolore the impact of previously mentioned concepts in a duopoly market, our present paper considers a consistent framework, making use of a product differentiation base model. We are discussing the market outcomes under quantity and price competition (Cournot & Bertrand simultaneous moves scenarios) but also sequential moves output competition (Stackelberg duopoly). We are also presenting a numerical simulation analyze that can be efficiently used to explore the model properties under the assumption of products differentiation degree variation. The models utilize both numerical and graphical presentation.

Key words: Stackelberg equilibrium, Stackelberg model, Cournot model, oligopoly, stability.
J.E.L. classification: C72, D01, D43, L13

1. Introduction

Oligopoly represents one of the main forms of imperfect competition. During various oligopoly theories over the years, three standard textbook models have been developed aiming to explain both economic output and pricing related decisions: Cournot model (1838), Bertrand model (1883) and the last but not the least Stackelberg model (1934). Due to the existence of various types of firms interaction as well as the complex nature of their interdependences, the use of only one oligopoly model is not adequate, therefore the next paragraphs will consider all three above mentioned models. At least four main aspects should be considered at the start of any market structure analysis: the decision of competing in price or output terms (very important topic in industrial organization), the timing of the competitors movement (sequential or simultaneous), the products typology (differentiated or homogenous) as well as the existence of a hierarchical structure. Subject to the above mentioned factors mix, the market performance and profit distribution will be significantly different.

In both Cournot and Betrand models, the players are choosing their strategy simultaneously, but Cournot player set his output level (price beeing determined by some unspecified agent hence market demand match the aggregate offer), whilst Bertrand player establish its selling price (firms being constrained to immediately meet the resulting customer demand). On the other side Stackelberg model is an hierachical model with firms choosing their output level sequentially. The leader (the sophisticated firm) takes into account its ability to manipulate his rival's output, whilst the follower (the naïve firm) adopt a Cournot behavior, considering the other firm's output level fixed.

2. Literature review

The current literature comparing the two equilibria in terms of output (Cournot & Stackelberg) is abundant. The classic conclusion is that the Stackelberg equilibrium is more efficient, total surplus being higher in the sequential game scenario, as per Boyer and Moreaux (1986 & 1987), Daughety (1990); Robson (1990), Albaek (1990), Anderson and Engers, (1992), Amir and Grilo, (1999), Ino and Matsumura, (2012) papers. Other researchers approach was simultaneous game scenarios comparison (Cheng, 1985; Judd, 1989; Symeonidis, 2003; Haraguchi & Matsumura, 2015; etc.). However, the aggregate analyze, including all the three previously mentioned models, has received scant attention in the current literature.

In the current paper, a differentiated products scenario is considered, based on which we are trying to explain Cournot, Bertrand and also Stackelberg static behavior, highlighting some interesting aspects such as market surviving potential, firm equilibrium and the impact of product differentiation on Nash equilibrium/subgame perfect equilibrium theory. The originality of this paper arises from the ideea of bringing together and comparing simultaneously, all three mentioned models, not just theorethically, but also making use of numerical simulation. The principles of the related mathematic model are also described below.

3. Research methodology

The scenario used in the paper is one with plenty consumers but only two producers of differentiated goods. What the consumers are actually targeting to maximize, is their own satisfaction, described as the difference between own utility function and the spendings involved by the required product amounts purchasing (no budgetary constraints are considered):

$$S = U(q_1, q_2) - \sum_{i=1}^{2} p_i\, q_i \qquad (1)$$

The choosen utility function belongs to quadratic class (non-linear type), having separable variables, also assuming strict concavity. The last hypothesis involves double derivability, the existence of a second order derivate as well as its negativity.

$$U(q_1, q_2) = a q_1 + a q_2 - \frac{b q_1^2 + 2 d q_1 q_2 + b q_2^2}{2} \qquad (2)$$

where $a > 0, b > 0,\ d > 0$ (substitute products). Considering $b > d$, an imperfect substitutability is reflected, whilst setting $b = d$ a homogenous product scenario is assumed.

The starting point in duopoly direct but also inverse demand functions calculation, is represented by the derivation of the consumer satisfaction function. Their expressions are determinated as follows:

$$\boldsymbol{\frac{\partial S}{\partial q_1} = p_1 = a - bq_1 - dq_2 \Rightarrow q_1 = \frac{a - p_1 - dq_2}{b}} \qquad (3)$$

$$\frac{\partial S}{\partial q_2} = p_2 = a - bq_2 - dq_1 \Rightarrow q_2 = \frac{a - p_2 - dq_1}{b} \qquad (4)$$

Applying substitution methodology, will lead to:

$$q_1 = \frac{a(b-d) - bp_1 + dp_2}{b^2 - d^2} \quad (5) \qquad q_2 = \frac{a(b-d) - bp_2 + dp_1}{b^2 - d^2} \quad (6)$$

a system similar to those before used by Dixit (1979), Singh &Vives (1984), Imperato et all (2004), Tremblay (2011).

It can be noted the necessity that $b > d$ at this stage.

The production costs are further considered different, expressed by linear functions ($C_1 = c_1 * q_1$, $C_2 = c_2 * q_2$), also matching marginal costs. Based on these assumptions, the profit function become:

$$\pi_i = (p_i - c_i)q_i \,, (\forall) i = \overline{1,2} \qquad (7)$$

and further

$$\pi_1 = (p_1 - c_1)q_1 = aq_1 - \mathrm{b}q_1^2 - dq_1q_2 - c_1q_1 \quad (8) \qquad \pi_1 = \frac{\mathrm{a(b-d) - bp_1 + dp_2}}{b^2 - d^2}(p_1 - c_1) \quad (9)$$

$$\pi_2 = (p_2 - c_2)q_2 = aq_2 - \mathrm{b}q_2^2 - dq_1q_2 - c_2q_2 \quad (10) \qquad \pi_2 = \frac{\mathrm{a(b-d) - bp_2 + dp_1}}{b^2 - d^2}(p_2 - c_2) \quad (11)$$

The market output level/selling price depends on the type of two firms interaction. If the duopolists choose to adopt an output strategy, deciding to move simultaneously, without knowing his rival answer, we are dealing with a Cournot behavior. By solving the profit maximization problem in terms of output, we can find out the players best response functions. Further application of systems solving substituting method leads to the of the Nash equilibrium output values:

$$\begin{cases} \frac{\partial \pi_1}{\partial q_1} = a - 2bp_1 - dq_2 - c_1 = 0 \\ \frac{\partial \pi_2}{\partial q_2} = a - 2bp_2 - \mathrm{d}q_1 - c_2 = 0 \end{cases} \Rightarrow \begin{cases} q_1 = \frac{a - c_1 - dq_2}{2\mathrm{b}} \\ q_2 = \frac{a - c_2 - dq_1}{2b} \end{cases} \Rightarrow q_1 = \frac{a - c_1 - d\frac{a - c_2 - dq_1}{2b}}{2\mathrm{b}} = \frac{2ab - 2\mathrm{b}c_1 - \mathrm{a}d + dc_2 + d^2q_1}{4b^2}$$

$$q_1(4b^2 - d^2) = a(2b - d) - 2bc_1 + dc_2 \Rightarrow q_1 = \frac{a}{2\mathrm{b+d}} - \frac{2bc_1 - dc_2}{4b^2 - d^2} \quad (12) \Rightarrow$$

$$q_2 = \frac{a - c_2 - d\left(\frac{a}{2\mathrm{b+d}} - \frac{2bc_1 - dc_2}{4b^2 - d^2}\right)}{2\mathrm{b}} = \frac{2ab - 2bc_2 - ad + dc_1}{2b} = \frac{a(2\mathrm{b-d})}{4b^2 - d^2} - \frac{2bc_2 - dc_1}{4b^2 - d^2} = \frac{a}{2\mathrm{b+d}} - \frac{2bc_2 - dc_1}{4b^2 - d^2} \quad (13)$$

The corresponding prices, will result immediately:

$$p_1 = \mathrm{a} - \mathrm{b}\left(\frac{a}{2\mathrm{b+d}} - \frac{2bc_1 - dc_2}{4b^2 - d^2}\right) - \mathrm{d}\left(\frac{a}{2\mathrm{b+d}} - \frac{2bc_2 - dc_1}{4b^2 - d^2}\right) = \frac{ab}{2\mathrm{b+d}} + \frac{c_1(2b^2 - d^2) + bdc_2}{4b^2 - d^2} \quad (14)$$

$$p_2 = \mathrm{a} - \mathrm{b}\left(\frac{a}{2\mathrm{b+d}} - \frac{2bc_2 - dc_1}{4b^2 - d^2}\right) - \mathrm{d}\left(\frac{a}{2\mathrm{b+d}} - \frac{2bc_1 - dc_2}{4b^2 - d^2}\right) = \frac{ab}{2\mathrm{b+d}} + \frac{c_2(2b^2 - d^2) + bdc_1}{4b^2 - d^2} \quad (15)$$

and finally the profits level will be determinated

$$\pi_1 = (p_1 - c_1)\, q_1 = \left(\frac{ab}{2\mathrm{b+d}} + \frac{c_1(2b^2 - d^2) + bdc_2}{4b^2 - d^2} - c_1\right)\left(\frac{a}{2\mathrm{b+d}} - \frac{2bc_1 - dc_2}{4b^2 - d^2}\right) = \frac{a^2b}{(2b+d)^2} - \frac{2\mathrm{ab}(2bc_1 - dc_2)}{(2b+d)(4b^2 - d^2)} + \frac{\mathrm{b}(2bc_1 - dc_2)^2}{(4b^2 - d^2)^2} \quad (16)$$

$$\pi_2 = (p_2 - c_2)\, q_2 = \left(\frac{ab}{2\mathrm{b+d}} + \frac{c_2(2b^2 - d^2) + bdc_1}{4b^2 - d^2} - c_2\right)\left(\frac{a}{2\mathrm{b+d}} - \frac{2bc_2 - dc_1}{4b^2 - d^2}\right) = \frac{a^2b}{(2b+d)^2} - \frac{2\mathrm{ab}(2bc_2 - dc_1)}{(2b+d)(4b^2 - d^2)} + \frac{\mathrm{b}(2bc_2 - dc_1)^2}{(4b^2 - d^2)^2} \quad (17)$$

If the duopolists decide to compete in terms of price instead, their action path beeing also simultaneously manifested, the Bertrand scenario is assumed . First order condition for the profit expression, represent the starting point in the determination of the Nash equilibrium price values, all the related mathematic calculations beeing further described :

$$\begin{cases} \frac{\partial \pi_1}{\partial p_1} = \frac{\mathrm{a(b-d)-bp_1+dp_2}}{b^2-d^2} - \frac{\mathrm{b(p_1-}c_1)}{b^2-d^2} = 0 \\ \frac{\partial \pi_2}{\partial p_2} = \frac{\mathrm{a(b-d)-bp_2+dp_1}}{b^2-d^2} - \frac{\mathrm{b(p_2-}c_2)}{b^2-d^2} = 0 \end{cases} \Rightarrow \begin{cases} a(b-d) + bc_1 + dp_2 - 2b\,p_1 = 0 \\ a(b-d) + b\,c_2 + dp_1 - 2b\,p_2 = 0 \end{cases} \Rightarrow$$

$$\begin{cases} 2b\,p_1 - dp_2 = a(b-d) + b\,c_1 \\ -dp_1 + 2bp_2 = a(b-d) + b\,c_2 \end{cases}$$

The equations system solution is $p_1 = \frac{a(b-d)}{\mathrm{2b-d}} + \frac{b(2bc_1+dc_2)}{4b^2-d^2}$ (18) , $p_2 = \frac{a(b-d)}{\mathrm{2b-d}} + \frac{b(2bc_2+dc_1)}{4b^2-d^2}$ (19)

By substituting and solving the new created equations system (q_1 and q_2 as unknowns), will obtain:

$$q_1 = \frac{ab}{\mathrm{(b+d)(2b-d)}} - \frac{b^2dc_2 + bc_1(d^2-2b^2)}{(4b^2-d^2)(d^2-b^2)} \qquad (20)$$

$$q_2 = \frac{ab}{\mathrm{(b+d)(2b-d)}} - \frac{b^2dc_1 + bc_2(d^2-2b^2)}{(4b^2-d^2)(d^2-b^2)} \qquad (21)$$

$$\begin{aligned}\pi_1 = (p_1 - c_1)\,q_1 &= \left(\frac{a(b-d)}{\mathrm{2b-d}} + \frac{b(2bc_1+dc_2)}{4b^2-d^2} - c_1\right)\left(\frac{ab}{\mathrm{(b+d)(2b-d)}} + \frac{b^2dc_2 + bc_1(d^2-2b^2)}{(4b^2-d^2)(d^2-b^2)}\right) \\ &= \frac{a^2b(b-d)}{\mathrm{(b+d)(2b-d)^2}} + \frac{\mathrm{a}b[bdc_2 + c_1(d^2-2b^2)]}{\mathrm{(b+d)(2b-d)}(4b^2-d^2)} - \frac{\mathrm{a}b^2(2bc_1 - dc_2)}{\mathrm{(b+d)(2b-d)}(4b^2-d^2)} \\ &+ \frac{b^2(4b^2dc_1c_2 + 2bd^2c_1^2 - 4b^3c_1^2 - bd^2c_2^2 - d^3c_1c_2)}{(4b^2-d^2)^2(d^2-b^2)} \qquad (22)\end{aligned}$$

$$\begin{aligned}\pi_2 = (p_2 - c_2)\,q_2 &= \left(\frac{a(b-d)}{\mathrm{2b-d}} + \frac{b(2bc_2+dc_1)}{4b^2-d^2} - c_2\right)\left(\frac{ab}{\mathrm{(b+d)(2b-d)}} + \frac{b^2dc_1 + bc_2(d^2-2b^2)}{(4b^2-d^2)(d^2-b^2)}\right) \\ &= \frac{a^2b(b-d)}{\mathrm{(b+d)(2b-d)^2}} + \frac{\mathrm{a}b[bdc_1 + c_2(d^2-2b^2)]}{\mathrm{(b+d)(2b-d)}(4b^2-d^2)} - \frac{\mathrm{a}b^2(2bc_2 - dc_1)}{\mathrm{(b+d)(2b-d)}(4b^2-d^2)} \\ &+ \frac{b^2(4b^2dc_1c_2 + 2bd^2c_2^2 - 4b^3c_2^2 - bd^2c_1^2 - d^3c_1c_2)}{(4b^2-d^2)^2(d^2-b^2)} \qquad (23)\end{aligned}$$

If a firms sequential moving scenario is preffered to the simultaneous moving one, we face a Stackelberg model. Assuming player 1 as the first mover, the main problem will be to maximize its profit level, considering his rival's subsequent move, which is not controlable but at list predictable. This issue can be solved by using backward induction. In the second stage, the follower chooses his output level in order to maximize profit, given the output choice of the leader. In the first stage instead, the leader chooses its profit maximizing output knowing exactly what his rival's answer is. All the related mathematic Appendix calculations, leads to the bellow mentioned subgame perfect equilibrium values:

$$q_1 = \frac{2ab - ad + dc_2 - 2bc_1}{4b^2 - 2d^2} \quad (24) \qquad q_2 = \frac{4ab^2 - ad^2 - 4b^2c_2 + d^2c_2 - 2abd + 2\mathrm{bd}c_1}{2b(4b^2 - 2d^2)} \quad (25)$$

$$\mathrm{p}_1 = \frac{2ab - ad + dc_2 + 2bc_1}{\mathrm{4b}} \quad (26) \qquad \mathrm{p}_2 = \frac{4ab^2 - ad^2 + 4b^2c_2 - 3d^2c_2 - 2abd + 2\mathrm{bdc}_1}{4(2b^2 - d^2)} \quad (27)$$

$$\pi_1 = \frac{(2ab - ad + dc_2 - 2bc_1)^2}{8b(2b^2 - d^2)} \quad (28) \quad \pi_2 = \frac{(4ab^2 - ad^2 - 4b^2c_2 + d^2c_2 - 2abd + 2\mathrm{bd}c_1)^2}{16b(2b^2 - d^2)^2} \quad (29)$$

Comparing the equilibrium values of both output strategy games (Cournot and Stackelberg), some interesting conclusions can be reached:

- The leader output level is higher in the Stackelberg scenario, knowing that the follower will respond by cutting its own;
- The leader / the follower charge lower prices than in Cournot game.
- Although duopolist's aggregate profits fall, the leader win extra profits by taking a greater market share. This is the first-mover undeniable advantage of the Stackelberg game.

Next paragraphs will analyze particular situations than can appear. First one is the $b = d$ scenario (perfect substitute products situation). The equilibrium values can be synthesized as per below table:

Table no. 1 Cournot/Bertrand/Stackelberg homogenous products equilibrium figures

Variable	$\mathbf{p_1}$	$\mathbf{p_2}$	$\mathbf{q_1}$	$\mathbf{q_2}$	$\boldsymbol{\pi_1}$	$\boldsymbol{\pi_2}$
Cournot	$\frac{a+c_1+c_2}{3}$	$\frac{a+c_1+c_2}{3}$	$\frac{a-2c_1+c_2}{3b}$	$\frac{a+c_1-2c_2}{3b}$	$\frac{(a-2c_1+c_2)^2}{9b}$	$\frac{(a+c_1-2c_2)^2}{9b}$
Bertrand	c	c	$\frac{a-c}{2b}$	$\frac{a-c}{2b}$	0	0
Stackelberg	$\frac{a+2c_1+c_2}{4}$	$\frac{a+2c_1+c_2}{4}$	$\frac{a-2c_1+c_2}{2b}$	$\frac{a+2c_1-3c_2}{4b}$	$\frac{(a-2c_1+c_2)^2}{8b}$	$\frac{(a+2c_1-3c_2)^2}{16b}$

Source: authors' calculations

Remarks:

- Making $c_1 < c_2$ assumption, two thirds of the Stackelberg leader reached output matches first player outcome in the simultaneous output game, latter result beeing higher than the Cournot second player / Stackelberg follower's output. If we decide to consider $c_1 > c_2$ hypothesis, an opposite rank can be considered. In the simultaneous price scenario instead, both players produce the same outcome.
- The highest level of charged price is triggered in the Cournot scenario, with sequential game strategy following closely. The Bertrand game instead, reveals the most interestig situation, both players charged price matching lowest possible value for mantaining economic rentability - the marginal cost (Bertrand paradox).
- Although the highest profit level is attained by the Stackelberg leader, aggregate value is maximized in the Cournot scenario. Bertrand strategy offers aggregate zero profit, as a Bertrand paradox consequence.
- In the simultaneous price scenario the related results are determinated by the simple fact that both players production costs should match. However, if a different situation can be found on the market, only the firm with lower cost is expected to survive (producing the perfectly competitive market output level) whilst the other one will leave the market.

The second highlighted scenario is the particular one when d = 0 (independent products). Once triggered, the firms connection will be lost, and it will no longer matter the type of the choosen strategy (a simple look at Table no. 2 values is good enough to highlight this aspect).

Table no. 2 Cournot/Bertrand/Stackelberg independent products equilibrium figures

Variable	$\mathbf{p_1}$	$\mathbf{p_2}$	$\mathbf{q_1}$	$\mathbf{q_2}$	$\boldsymbol{\pi_1}$	$\boldsymbol{\pi_2}$
Cournot	$\frac{a+c_1}{2}$	$\frac{a+c_2}{2}$	$\frac{a-c_1}{2b}$	$\frac{a-c_2}{2b}$	$\frac{(a-c_1)^2}{4b}$	$\frac{(a-c_2)^2}{4b}$
Bertrand	$\frac{a+c_1}{2}$	$\frac{a+c_2}{2}$	$\frac{a-c_1}{2b}$	$\frac{a-c_2}{2b}$	$\frac{(a-c_1)^2}{4b}$	$\frac{(a-c_2)^2}{4b}$
Stackelberg	$\frac{a+c_1}{2}$	$\frac{a+c_2}{2}$	$\frac{a-c_1}{2b}$	$\frac{a-c_2}{2b}$	$\frac{(a-c_1)^2}{4b}$	$\frac{(a-c_2)^2}{4b}$

Source: authors' calculations

4. Findings

The next paragraphs are trying to facilitate a better understanding of the three behavioral types described in the paper as well as impact when products substitutability degree starts to change. With this precise goal, we illustrate a differentiated products duopoly scenario, assuming specific values for model entrance parameters, as follows: a = 500, b=2, c_1 = 60, c_2 = 50. Once this hypothesis made, we consider the model's main parameter matching d = 1 value.

The demand functions will be obtained by substituting parameters values in (3) and (4); the result will be:

$$\frac{\partial \mathbf{U}}{\partial \mathbf{q_1}} = \mathbf{500 - 2q_1 - q_2 = p_1} \qquad \frac{\partial \mathbf{U}}{\partial \mathbf{q_2}} = \mathbf{500 - 2q_2 - q_1 = p_2}$$

$$q_1 = \frac{500 - p_1 - q_2}{2} \qquad q_2 = \frac{500 - p_2 - q_1}{2}$$

Further substituting quantities expressions in (5) and (6), inverse demand functions are revealed:

$$q_1 = \frac{500 - p_1 - \frac{500 - p_2 - q_1}{2}}{2} = \frac{1000 - 2p_1 - 500 + p_2 + q_1}{4} \rightarrow q_1 = 166,(6) - 0.(6)p_1 + 0.(3)p_2$$

$$q_2 = \frac{500 - p_2 - \frac{500 - p_1 - q_2}{2}}{2} = \frac{1000 - 2p_2 - 500 + p_1 + q_2}{4} \rightarrow q_2 = 166,(6) - 0.(6)p_2 + 0.(3)p_1$$

Consumer surplus can be calculated as the difference between own utility function and price for purchasing required product amounts, whilst aggregate surplus includes also the producers profits.

A complete situation with all three equilibrium scenarios values, is analyzed in the bellow table:

Table no. 3 Cournot/Bertrand/Stackelberg equilibrium figures in b=2 & d=1 scenario

Strategic variable	$\mathbf{p_1}$	$\mathbf{p_2}$	$\mathbf{q_1}$	$\mathbf{q_2}$	$\boldsymbol{\pi_1}$	$\boldsymbol{\pi_2}$	$\mathbf{S_c}$	$\mathbf{S_t}$
Cournot model	234.7	231.3	87.3	90.7	15254.2	16440.9	23765.8	55460.9
Bertrand model	205.3	201.3	96.9	100.9	13693.6	14931.6	29341.0	57966.2
Stackelberg model	223.8	228.2	93.6	89.1	15322.3	15880.2	25033.6	56236.1

Source: authors' calculations

The degree of product differentiation can be easily adjusted, and we face now a much differentiated products scenario, as d = 0.5. This time, the proper substitutions led to the following result:

$$\frac{\partial \mathbf{U}}{\partial \mathbf{q_1}} = \mathbf{500 - 2q_1 - 0.5q_2 = p_1} \qquad \frac{\partial \mathbf{U}}{\partial \mathbf{q_2}} = \mathbf{500 - 2q_2 - 0.5q_1 = p_2}$$

$$q_1 = \frac{500 - p_1 - 0.5q_2}{2} \qquad q_2 = \frac{500 - p_2 - 0.5q_1}{2}$$

whilst inverse demand functions became:

$$q_1 = \frac{500 - p_1 - \frac{500 - p_2 - 0.5q_1}{4}}{2} = \frac{2000 - 4p_1 - 500 + p_2 + 0.5q_1}{8} \rightarrow q_1 = 200 - 0.5(3)p_1 + 0.1(3)p_2$$

$$q_2 = \frac{500 - p_2 - \frac{500 - p_1 - 0.5q_2}{4}}{2} = \frac{2000 - 4p_2 - 500 + p_1 + 0.5q_2}{8} \rightarrow q_2 = 200 - 0.5(3)p_2 + 0.1(3)p_1$$

Consumer/aggregate surplus and all other equilibrium values, can be further observed:

Table no. 4 Cournot/Bertrand/Stackelberg equilibrium figures in b=2 & d=0.5 scenario

Strategic variable	$\mathbf{p_1}$	$\mathbf{p_2}$	$\mathbf{q_1}$	$\mathbf{q_2}$	$\boldsymbol{\pi_1}$	$\boldsymbol{\pi_2}$	$\mathbf{S_c}$	$\mathbf{S_t}$
Cournot model	254.9	250.6	97.5	100.3	18997.0	20127.2	24450.6	63574.8
Bertrand model	247.9	243.5	100.2	103.2	18741.9	19885.7	25867.8	64495.4
Stackelberg model	251.9	250.2	99.0	100.1	19001.8	20048.4	24789.2	63839.4

Source: authors' calculations

Finally, we consider a high homogeneity product degree scenario, reflected by a d = 1.5 value. Demand function expression will be:

$$\frac{\partial \mathbf{U}}{\partial \mathbf{q_1}} = \mathbf{500 - 2q_1 - 1.5q_2 = p_1} \qquad \frac{\partial \mathbf{U}}{\partial \mathbf{q_2}} = \mathbf{500 - 2q_2 - 1.5q_1 = p_2}$$

$$q_1 = \frac{500 - p_1 - 1.5q_2}{2} \qquad q_2 = \frac{500 - p_2 - 1.5q_1}{2}$$

whilst inverse demand functions became:

$$q_1 = \frac{500 - p_1 - 3\frac{500 - p_2 - 1.5q_1}{4}}{2} = \frac{2000 - 4p_1 - 1500 + 3p_2 + 4.5q_1}{8} \rightarrow q_1 = 142{,}86 - 1.14p_1 + 0.86p_2$$

$$q_2 = \frac{500 - p_2 - 3\frac{500 - p_1 - 1.5q_2}{4}}{2} = \frac{2000 - 4p_2 - 1500 + 3p_1 + 4.5q_2}{8} \rightarrow q_2 = 142.(857142) - 1.(142857)p_2 + 0.(857142)p_1$$

Consumer surplus and all other equilibrium values, can be synthesized as per below:

Table no. 5 Cournot/Bertrand/Stackelberg equilibrium figures in b=2 & d=1.5 scenario

Strategic variable	p_1	p_2	q_1	q_2	π_1	π_2	S_c	S_t
Cournot model	217.8	215.8	78.9	82.9	12453.3	13747.83	22914.0	49115.1
Bertrand model	145.8	142.2	98.1	105.4	7453.9	8849.5	36216.9	52520.3
Stackelberg model	195.6	204.2	94.3	77.1	12795.9	11894.9	25763.0	50453.8

Source: authors' calculations

5. Conclusions

In all three models *d* parameter respresents the degree of product differentiation. With particular scenarios of perfectly substitutes / independent products already discussed, we further consider as *d* value interest area, the mathematic interval [0,10; 1.90] (a very poor / high product differentiation degree, trigger significantly closer scenarios to the homogenous products / monopoly solutions, then will be excluded from our current analyze).

As the *d* value increases, aproaching to the *b* parameter value (homogenous products case), the results are significantly different. Models based on a strategy in terms of output, offer a price equilibrium solution, far enough above marginal costs, despite the fact that the profits decline. In the Bertrand game instead, the outcome level decreases, getting close to the marginal costs area. Previous arguments can be successfully used in the tentative of making firms aware of the product differentiation importance, especially in the price competition scenario.

Turning our attention to the Stackelberg model, the paper is trying to explain what exactly happens with the strategic choice of the leader, when the product differentiation degree changes. In a homogeneous products scenario, the leader will decide to produce an output level, lower but very close to the monopoly level. If d parameter start to decrease, the leader will be constrained to reduce his produced quantity. As is well known, a small change of product differentiation degree will impact duopolists best reply functions. Focusing on the leader, two contrary effects will manifest. Although for any given level of follower's output, the leader's intention is to produce more, an increase in the rival's produced output compel the leader to produce less. The latter effect dominates initially, but at a certain moment, the former effect must take over, the produced amount trend will be inversed, rising toward the monopoly solution, as d parameter value approaches to zero.

We can further expand by using mathematic principles to also prove the previously reached conclusions. Maintaining the direction, we are highlighting the above simulation figures, their connection with the equilibrium values functions monotony, induced by d parameter variation. This way, last results can be rephrased as follows:

- In a simultaneous output competition game (Cournot game), initial scenario (d=1) ofer better results than in the high homogeneity scenario (d=1.5), but worse than in the poor substitutability situation (d=0.5). Equilibrium quantity, price and also profits follows decreasing trends, as the homogenity degree starts to increase (induced by first order derivatives negativity for the entire [0.10;1.90] interval). Consumer satisfaction is affected in the same manner and obviously the aggregate market surplus confirm the trend.
- In the price competition game (Betrand game), different trends can be observed: prices, profits and also aggregate market surplus fall, as the degree of product differentiation decrease (second scenario reveals the best result). The equilibrium quantity „hide" a scenarios mixture, decreasing trend validity maintaining as long as d value keeps outside of (1.09;1.69) area (for the first player), respectevely lower than 0.94 (for the second player). In any other scenario, the trend will be inversed. Consumer surplus instead, highlights an increasing trend, the total amount they have to pay for purchasing the required product quantity, lowering as the product homogeneity increase.
- In the sequential game (Stackelberg game), prices and profits fall as the substitutability degree increases, as well as the follower produced output (second scenario offers the best result whilst third scenario, the worst). The leader's equilibrium output level instead, follows a decreasing trend, as d parameter keeps lower than 1,21 value; after that, once the increase begins to manifest, the output level get close to the monopoly solution. Consumer surplus level registers an increasing trend on the interest area, but not good enough to affect the market aggregate surplus evolution, strongly influenced by the producers profits fall, once the products differentiation degree has started to increase.

6. References

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Appendix

$$\left.\begin{array}{r} a - 2bp_2 - \mathrm{d}p_1 - c_2 = 0 \Rightarrow q_2 = \frac{a-c_2-dq_1}{2b} \\ \pi_1 = aq_1 - b\mathrm{q}_1^2 - dq_1q_2 - c_1q_1 \end{array}\right\} \Rightarrow \pi_1 = aq_1 - b\mathrm{q}_1^2 - dq_1\frac{a-c_2-dq_1}{2b} - c_1q_1$$

$$2b\pi_1 = 2abq_1 - 2b^2\mathrm{q}_1^2 - adq_1 + dq_1c_2 + d^2\mathrm{q}_1^2 - 2bc_1q_1 \Rightarrow \pi_1 = q_1\frac{2ab - ad + dc_2 - 2bc_1}{2\mathrm{b}} - \frac{2b^2 - d^2}{2\mathrm{b}}\mathrm{q}_1^2$$

$$\frac{\partial \pi_1}{\partial q_1} = 0 \Rightarrow \frac{2ab - ad + dc_2 - 2bc_1}{2\mathrm{b}} - \frac{4b^2 - 2d^2}{2\mathrm{b}}q_1 = 0 \Rightarrow q_1 = \frac{2ab - ad + dc_2 - 2bc_1}{4b^2 - 2d^2}$$

$$q_2 = \frac{a - c_2 - d\frac{2ab - ad + dc_2 - 2bc_1}{4b^2 - 2d^2}}{2b} = \frac{4ab^2 - 2ad^2 - 4b^2c_2 + 2d^2c_2 - 2abd + 2bdc_1 + ad^2 - d^2c_2}{2b(4b^2 - 2d^2)}$$

$$\Rightarrow q_2 = \frac{4ab^2 - ad^2 - 4b^2c_2 + d^2c_2 - 2abd + 2bdc_1}{2b(4b^2 - 2d^2)}$$

$$\mathrm{p}_1 = \mathrm{a} - \mathrm{bq}_1 - \mathrm{dq}_2 = a - b\frac{2ab - ad + dc_2 - 2bc_1}{4b^2 - 2d^2} - d\frac{4ab^2 - ad^2 - 4b^2c_2 + d^2c_2 - 2abd + 2bdc_1}{2b(4b^2 - 2d^2)}$$

$$= \frac{2ab(4b^2 - 2d^2) - 2b^2(2ab - ad + dc_2 - 2bc_1) - d(4ab^2 - ad^2 - 4b^2c_2 + d^2c_2 - 2abd + 2bdc_1)}{2b(4b^2 - 2d^2)}$$

$$= \frac{8ab^3 - 4abd^2 - 4ab^3 + 2ab^2d - 2b^2dc_2 + 4b^3c_1 - 4ab^2d + ad^3 + 4b^2dc_2 - d^3c_2 + 2abd^2 - 2bd^2c_1}{2b(4b^2 - 2d^2)}$$

$$= \frac{4ab^3 - 2abd^2 - 2ab^2d + 2b^2dc_2 + 4b^3c_1 + ad^3 - d^3c_2 - 2bd^2c_1}{2b(4b^2 - 2d^2)}$$

$$= \frac{2ab(2b^2 - d^2) - ad(2b^2 - d^2) - dc_2(2b^2 - d^2) + 2\mathrm{b}c_1(2b^2 - d^2)}{4b(2b^2 - d^2)} = \frac{2ab - ad + dc_2 + 2bc_1}{4\mathrm{b}}$$

$$\mathrm{p}_2 = \mathrm{a} - \mathrm{bq}_2 - \mathrm{dq}_1 = a - b\frac{4ab^2 - ad^2 - 4b^2c_2 + d^2c_2 - 2abd + 2bdc_1}{2b(4b^2 - 2d^2)} - d\frac{2ab - ad + dc_2 - 2bc_1}{4b^2 - 2d^2}$$

$$= \frac{2ab(4b^2 - 2d^2) - 2\mathrm{bd}(2ab - ad + dc_2 - 2bc_1) - b(4ab^2 - ad^2 - 4b^2c_2 + d^2c_2 - 2abd + 2bdc_1)}{2b(4b^2 - 2d^2)}$$

$$= \frac{8ab^3 - 4abd^2 - 4ab^3 + 2abd^2 - 2bd^2c_2 + 4b^3c_2 + 2ab^2d - abd^2 - 2b^2dc_1 - 4ab^2d + 2abd^2 + 4b^2dc_1}{2b(4b^2 - 2d^2)}$$

$$= \frac{4ab^3 - 2abd^2 - bd^2c_2 + 4b^3c_2 - 2ab^2d + 2b^2dc_1 + 2abd^2 - 2bd^2c_2 - abd^2}{4b(2b^2 - d^2)}$$

$$= \frac{b(4ab^2 - 3d^2c_2 + 4b^2c_2 - 2\mathrm{abd} + 2\mathrm{bd}c_1 - ad^2)}{4b(2b^2 - d^2)} = \frac{4ab^2 - 3d^2c_2 + 4b^2c_2 - 2\mathrm{abd} + 2\mathrm{bd}c_1 - ad^2}{4(2b^2 - d^2)}$$

$$\pi_1 = (p_1 - c_1)q_1 = \left(\frac{2ab - ad + dc_2 + 2bc_1}{4\mathrm{b}} - c_1\right)\frac{2ab - ad + dc_2 - 2bc_1}{4b^2 - 2d^2} = \frac{(2ab - ad + dc_2 - 2bc_1)^2}{8b(2b^2 - d^2)}$$

$$\pi_2 = \left(\frac{4ab^2 - 3d^2c_2 + 4b^2c_2 - 2\mathrm{abd} + 2\mathrm{bd}c_1 - ad^2}{4(2b^2 - d^2)} - c_2\right)\frac{4ab^2 - ad^2 - 4b^2c_2 + d^2c_2 - 2abd + 2bdc_1}{2b(4b^2 - 2d^2)}$$

$$= \frac{(4ab^2 - ad^2 - 4b^2c_2 + d^2c_2 - 2abd + 2bdc_1)^2}{16b(2b^2 - d^2)^2} = (p_2 - c_2)q_2$$